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EFFECTS OF VIBRATIONAL RELAXATION OF MULTI-ATOMIC MOLECULES ON STAGNATION HEAT TRANSFER

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Abstract—The flow and temperature fields of multi-atomic gas near the stagnation of a cylinder are theoretically studied, taking molecular vibrational relaxation into account, and putting importance on heat transfer performances. Based on the continuum equations, the velocity and temperature jumps of translational, rotational and vibrational degrees are considered as the boundary conditions and effects of the vibrational relaxation for subsonic flow are discussed. The total heat transferred to the surface is the sum of heats due to the translational-rotational temperature and the vibrational temperature. The ratio of the heats depends much on the ratio of the flow characteristic time to the relaxation time of molecule, and consequently the total heat is much influenced by this ratio of the characteristic times. The effect of the flow-relaxation time on heat transfer is more remarkable in the vibrational nonequilibrium than in the thermal equilibrium main flow. Even in a continuum flow of Knudsen number of about 10^{-4}, the effect of vibrational relaxation on stagnation heat transfer is still appreciable.

NOMENCLATURE

C,	ratio of isochoric vibrational specific heat to isochoric specific heat without vibrational contribution $C = \hat{C}_{vv}/\hat{C}_{vt,r}$;
$\hat{C}_p$,	isobaric specific heat $\hat{C}_g = \hat{C}_{pt,r} + \hat{C}_{vv}$;
$\hat{C}_v$,	isochoric specific heat $\hat{C}_v = \hat{C}_{vt,r} + \hat{C}_{vv}$;
$\hat{D}$,	diameter of cylinder;
f,	momentum reflection coefficient of molecule;
K,	ratio of characteristic flow time to vibrational relaxation time;
Kn,	Knudsen number, $Kn = \hat{l}/\hat{D}$;
$\hat{l}$,	mean free path of molecule;
$\hat{p}$,	thermodynamic pressure;
p,	nondimensional thermodynamic pressure $p = \hat{p}/\hat{\rho}_\infty \hat{V}_\infty^2$;
Pr,	Prandtl number $Pr = \hat{\mu}\hat{C}_p/(\hat{\lambda}_{t,r} + \hat{\lambda}_v)$;
Pr_{I},	Prandtl number without vibrational contribution $Pr_{\mathrm{I}} = \hat{\mu}\hat{C}_{vt,r}/\hat{\lambda}_{t,r}$;
Pr_{II},	Prandtl number due to vibrational degree $Pr_{\mathrm{II}} = \hat{\mu}\hat{C}_{vv}/\hat{\lambda}_v$;
$\hat{r}$,	radial coordinate;
r,	nondimensional radial coordinate $r = \hat{r}/\hat{D}$;
$\hat{R}$,	specific gas constant;
Re,	Reynolds number $Re = \hat{\rho}_\infty \hat{V}_\infty \hat{D}/\hat{\mu}$;
$\hat{t}$,	time;
t,	nondimensional time $t = \hat{t} \cdot \hat{V}_\infty/\hat{D}$;
$\hat{T}$,	temperature;
T,	nondimensional temperature $T = \hat{T}\hat{C}_{pt,r}/\hat{V}_\infty^2$;
$\hat{T}_{t,r}$,	temperature of translational and rotational degrees;
$\hat{T}_v$,	temperature of vibrational degree;
$\hat{u}$,	circumferential velocity;
u,	nondimensional circumferential velocity $u = \hat{u}/\hat{V}_\infty$;
$\hat{v}$,	radial velocity;
v,	nondimensional radial velocity $v = \hat{v}/\hat{V}_\infty$;
$\alpha_{t,r}$,	accommodation coefficient of translational and rotational temperature;
α_v,	accommodation coefficient of vibrational temperature;
γ,	specific heat ratio $\gamma = \hat{C}_p/\hat{C}_v$;
γ_f,	specific heat ratio for frozen vibrational degree $\gamma_f = \hat{C}_{pt,r}/\hat{C}_{vt,r}$;
$\hat{\lambda}_{t,r}$,	translational and rotational thermal conductivity $\hat{\lambda}_{t,r} = (5\hat{\mu}\hat{C}_{vt}/2) + (6\hat{\mu}\hat{C}_{vr}/5)$;
$\hat{\lambda}_v$,	vibrational thermal conductivity $\hat{\lambda}_v = (6/5)\hat{\mu}\hat{C}_{vv}$;
$\lambda_{t,r}$,	nondimensional translational and rotational thermal conductivity $\lambda_{t,r} = \hat{\lambda}_{t,r}/(\hat{\lambda}_{t,r} + \hat{\lambda}_v)$;
λ_v,	nondimensional vibrational thermal conductivity $\lambda_v = \hat{\lambda}_v/(\hat{\lambda}_{t,r} + \hat{\lambda}_v)$;
$\hat{\mu}$,	viscosity;
$\hat{\rho}$,	density;
ρ,	nondimensional density $\rho = \hat{\rho}/\hat{\rho}_\infty$;
$\hat{\tau}$,	vibrational relaxation time.

Subscripts

t, r,	translation and rotation;
v,	vibration;
w,	wall quantities;
∞,	free stream quantities.

Superscripts

$\hat{}$,	dimensional quantities.

1. INTRODUCTION

SINCE the oil crisis in 1973, for effective utilizations of waste heat and thermal energy of hot brine from geothermal wells and in the ocean thermal energy conversion, Rankine cycle in low or middle temperature region has been attracting attention. Multi-atomic molecules such as freons, CO_2, NH_3, isobutane etc. have been investigated as working fluids for this purpose, as they have high vapor pressure at rather low temperatures. Some of these multi-atomic gases have comparatively long molecular vibrational relaxation time and their vibrational relaxation properties have been studied for the interests of chemical physics or chemical gas lasers [1], but no report of effects of vibrational relaxation of these multi-atomic gases on heat transfer has been made.

On the other hand, effects of molecular vibrational relaxation of a binary gas such as N_2 or O_2 or their mixture on heat transfer have been studied so far, mainly in the interests of stagnation heat transfer of blunt-nosed re-entry vehicles in the upper atmosphere.

In these studies, Jain and Adimurthy [2] have analysed the flow and temperature fields for the adiabatic wall case. Kao [3] analysed the viscous flow by applying the approximation of local similarity to the Navier–Stokes equations with the velocity and temperature jumps at the surface, but they did not take the vibrational relaxation into account. Tong [4] and Schubert [5] made theoretical investigations on stagnation flow and heat transfer behind a shock wave at hypersonic speeds and discussed effects of dissociational and vibrational relaxations, but both papers treated stagnation flows behind the shock wave downstream a hypersonic flow.

The features of this paper are different from the above papers on the following three points. First, cases for multi-atomic gases other than N_2 and O_2 have been treated. In the case of former gases, the ratio of the flow characteristic time to the vibrational relaxation time varies with molecules, and the ratio is taken as an important parameter in this paper. Secondly, the two main flows featured in this paper are discussed: one is a vibrational nonequilibrium main flow behind a weak shock and the other is an equilibrium main flow accompanied with the nonequilibrium region near the stagnation point. The two cases that are taken up refer to flows in components of organic Rankine cycle. Thirdly, dependence on effects of vibrational relaxation on Reynolds number is investigated, taking into consideration the variation of field properties in the circumferential direction.

2. FUNDAMENTAL EQUATIONS

The coordinates for the analysis of flow and temperature fields are shown in Fig. 1 where $\hat{V}_\infty$, $\hat{T}_{t,r\infty}$ and $\hat{T}_{v\infty}$ are the velocity, the translational temperature and the vibrational temperature, respectively. A cylinder of the diameter $\hat{D}$ is set in the flow, and the flow and temperature fields near the stagnation point are analysed under the following conditions. (1) The flow is steady, compressible, two-dimensional viscous. (2) The physical properties such as $\hat{C}_{pt,r}$, $\hat{C}_{vv}$, $\hat{\lambda}_{t,r}$, $\hat{\lambda}_v$, $\hat{\mu}$ are independent upon temperature, since as the temperature regions concerned are rather narrow. (3) Gases are ideal. (4) In a nonequilibrium state, the translational and rotational temperatures are always the same, while the vibrational temperature is different from those. (5) The velocity and temperature jumps at the surface are taken into account, but the accommodation coefficient for the vibrational temperature is very small compared with that of the translational-rotational temperature.

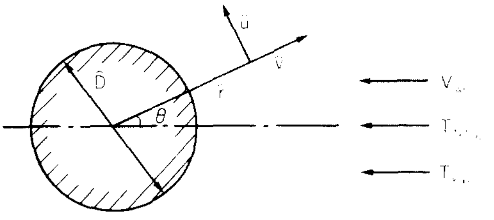

FIG. 1. Coordinate system.

In the following analysis the effect of curvature of the surface should not be neglected, as the influence of the stagnation point spreads widely upstream of it. Under the assumptions cited above, the fundamental equations are introduced.

Using the coordinate system shown in Fig. 1, the equation for mass is given as:

$$\frac{1}{\hat{r}}\frac{\partial}{\partial \hat{r}}(\hat{r}\hat{\rho}\hat{v}) + \frac{1}{\hat{r}}\frac{\partial}{\partial \theta}(\hat{\rho}\hat{u}) = 0. \tag{1}$$

The equations for momentum in the radial and circumferential directions are as shown by equations (2) and (3)

$$\begin{aligned}\hat{\rho}\hat{v}\frac{\partial \hat{v}}{\partial \hat{r}} + \frac{\hat{\rho}\hat{u}}{\hat{r}}\frac{\partial \hat{v}}{\partial \theta} - \frac{\hat{\rho}\hat{u}^2}{\hat{r}} &= -\frac{\partial \hat{p}}{\partial \hat{r}} + \frac{1}{3}\hat{\mu}\frac{\partial}{\partial \hat{r}}\left(\frac{1}{\hat{r}}\frac{\partial}{\partial \hat{r}}(\hat{r}\hat{v}) + \frac{1}{\hat{r}}\frac{\partial \hat{u}}{\partial \theta}\right) \\ &\quad + \hat{\mu}\left(\frac{\partial^2 \hat{v}}{\partial \hat{r}^2} + \frac{1}{\hat{r}}\frac{\partial \hat{v}}{\partial \hat{r}} + \frac{1}{\hat{r}^2}\frac{\partial^2 \hat{v}}{\partial \theta^2} - \frac{\hat{v}}{\hat{r}^2} - \frac{2}{\hat{r}^2}\frac{\partial \hat{u}}{\partial \theta}\right)\end{aligned} \tag{2}$$

$$\begin{aligned}\hat{\rho}\hat{v}\frac{\partial \hat{u}}{\partial \hat{r}} + \frac{\hat{\rho}\hat{u}}{\hat{r}}\frac{\partial \hat{u}}{\partial \theta} + \frac{\hat{\rho}\hat{v}\hat{u}}{\hat{r}} &= -\frac{1}{\hat{r}}\frac{\partial \hat{p}}{\partial \theta} + \frac{1}{3}\hat{\mu}\frac{1}{\hat{r}}\frac{\partial}{\partial \theta}\left(\frac{1}{\hat{r}}\frac{\partial}{\partial \hat{r}}(\hat{r}\hat{v}) + \frac{1}{\hat{r}}\frac{\partial \hat{u}}{\partial \theta}\right) \\ &\quad + \hat{\mu}\left(\frac{\partial^2 \hat{u}}{\partial \hat{r}^2} + \frac{1}{\hat{r}}\frac{\partial \hat{u}}{\partial \hat{r}} + \frac{1}{\hat{r}^2}\frac{\partial^2 \hat{u}}{\partial \theta^2} + \frac{2}{\hat{r}^2}\frac{\partial \hat{v}}{\partial \theta} - \frac{\hat{u}}{\hat{r}^2}\right).\end{aligned} \tag{3}$$

The equation for total energy is

$$\begin{aligned}&\hat{\rho}\hat{v}\frac{\partial}{\partial \hat{r}}(\hat{C}_{vt,r}\hat{T}_{t,r} + \hat{C}_{vv}\hat{T}_v) + \frac{\hat{\rho}\hat{u}}{\hat{r}}\frac{\partial}{\partial \theta}(\hat{C}_{vt,r}\hat{T}_{t,r} + \hat{C}_{vv}\hat{T}_v) \\ &\quad = -\hat{p}\left(\frac{\hat{v}}{\hat{r}} + \frac{\partial \hat{v}}{\partial \hat{r}} + \frac{1}{\hat{r}}\frac{\partial \hat{u}}{\partial \theta}\right) \\ &\qquad + \hat{\lambda}_{t,r}\left(\frac{\partial^2 \hat{T}_{t,r}}{\partial \hat{r}^2} + \frac{1}{\hat{r}}\frac{\partial \hat{T}_{t,r}}{\partial \hat{r}} + \frac{1}{\hat{r}^2}\frac{\partial^2 \hat{T}_{t,r}}{\partial \theta^2}\right) \\ &\qquad + \hat{\lambda}_v\left(\frac{\partial^2 \hat{T}_v}{\partial \hat{r}^2} + \frac{1}{\hat{r}}\frac{\partial \hat{T}_v}{\partial \hat{r}} + \frac{1}{\hat{r}^2}\frac{\partial^2 \hat{T}_v}{\partial \theta^2}\right)\end{aligned}$$

$$+\hat{\mu}\left[2\left\{\left(\frac{\partial \hat{v}}{\partial \hat{r}}\right)^2+\left(\frac{1}{\hat{r}}\frac{\partial \hat{u}}{\partial \theta}+\frac{\hat{v}}{\hat{r}}\right)^2\right\}+\left(\frac{1}{\hat{r}}\frac{\partial \hat{v}}{\partial \theta}+\frac{\partial \hat{u}}{\partial \hat{r}}-\frac{\hat{u}}{\hat{r}}\right)^2\right. \tag{4}$$

The equation for vibrational energy is

$$\hat{\rho}\hat{v}\frac{\partial}{\partial \hat{r}}(\hat{C}_{vv}\hat{T}_v)+\frac{\hat{\rho}\hat{u}}{\hat{r}}\frac{\partial}{\partial \theta}(\hat{C}_{vv}\hat{T}_v)$$
$$=\frac{\hat{\rho}\hat{C}_{vv}}{\hat{\tau}}(\hat{T}_{t,r}-\hat{T}_v)+\hat{\lambda}_v\left(\frac{\partial^2 \hat{T}_v}{\partial \hat{r}^2}+\frac{1}{\hat{r}}\frac{\partial \hat{T}_v}{\partial \hat{r}}+\frac{1}{\hat{r}^2}\frac{\partial^2 \hat{T}_v}{\partial \theta^2}\right) \tag{5}$$

with the relaxation time $\hat{\tau}$ of the molecule.

The equation of state is

$$\hat{p}=\hat{\rho}\hat{R}\hat{T}_{t,r}. \tag{6}$$

The two-dimensional flow and temperature fields in the vicinity of the stagnation region of a cylinder are going to be studied. The flow and temperature variables are expanded into the series of θ up to the second truncation as follows.

$$\hat{u}(\hat{r},\theta)=\hat{u}_1(\hat{r})\left(\theta-\frac{\theta^3}{6}\right) \tag{7}$$

$$\hat{v}(\hat{r},\theta)=\hat{v}_0(\hat{r})\left(1-\frac{\theta^2}{2}+\frac{\theta^4}{24}\right) \tag{8}$$

$$\hat{p}(\hat{r},\theta)=\hat{p}_0(\hat{r})+\theta^2\hat{p}_2(\hat{r}) \tag{9}$$

$$\hat{\rho}(\hat{r},\theta)=\hat{\rho}_0(\hat{r})+\theta^2\hat{\rho}_2(\hat{r}) \tag{10}$$

$$\hat{T}_{t,r}(\hat{r},\theta)=\hat{T}_{t,r0}(\hat{r})+\theta^2\hat{T}_{t,r2}(\hat{r}) \tag{11}$$

$$\hat{T}_v(\hat{r},\theta)=\hat{T}_{v0}(\hat{r}). \tag{12}$$

In the truncation equation of zero order of θ expansion for $\hat{T}_{t,r0}$, $\hat{T}_{t,r2}$ is included due to the thermal conduction term in the θ direction. The effect of the term of $\hat{T}_{t,r2}$ is limited in the layer near the surface and is accompanied by $\hat{p}_2$, which represents the influence of the curvature of the surface near the stagnation point. In this respect, $\hat{T}_{t,r2}$ can be analyzed corresponding to $\hat{p}_2$. However, the fundamental equations are of elliptic type and the exact solutions cannot be obtained without neglecting $\hat{T}_{t,r4}$, etc. Consequently, for this paper, $\hat{T}_{t,r2}$ was calculated under the following assumption. In the region so called the main stream, an adiabatic process is applied. In the region between the main stream and the surface, $\hat{T}_{t,r2}$ changes in a way similar to that of $\hat{T}_{t,r0}$ to satisfy the boundary condition at the surface of $\hat{T}_{t,r2}=0$.

$\hat{\rho}_2$ is expressed by use of $\hat{p}_2$ and $\hat{T}_{t,r2}$ from the equation of state, but the effect of $\hat{\rho}_2$ on momentum equation was found negligibly small through several trial calculations.

The terms of equal power of θ in the fundamental equation are summed up and made nondimensional by use of the following expressions:

$$v_0=\hat{v}_0/\hat{V}_\infty,\quad u_1=\hat{u}_1/\hat{V}_\infty,\quad \rho_0=\hat{\rho}_0/\hat{\rho}_\infty,$$
$$r=\hat{r}/\hat{D},\qquad t=\hat{t}\hat{V}_\infty/\hat{D},$$
$$p_i=\hat{p}_i/\hat{\rho}_\infty\hat{V}_\infty^2\ (i=0,2),\qquad T_j=\hat{T}_j\hat{C}_{pt,r}/\hat{V}_\infty^2\ (j=t,r,v)$$
$$\lambda_{t,r}=\hat{\lambda}_{t,r}/(\hat{\lambda}_{t,r}+\hat{\lambda}_v),\qquad \lambda_v=\hat{\lambda}_v/(\hat{\lambda}_{t,r}+\hat{\lambda}_v),$$
$$C=\hat{C}_{vv}/\hat{C}_{vt,r},\qquad \gamma_f=\hat{C}_{pt,r}/\hat{C}_{vt,r},$$
$$Re=\hat{\rho}_\infty\hat{V}_\infty\hat{D}/\hat{\mu},\qquad Pr_{\mathrm{I}}=\hat{\mu}\hat{C}_{vt,r}/\hat{\lambda}_{t,r},$$
$$Pr_{\mathrm{II}}=\hat{\mu}\hat{C}_{vv}/\hat{\lambda}_v,\qquad Pr=\hat{\mu}\hat{C}_p/(\hat{\lambda}_{t,r}+\hat{\lambda}_v)$$
$$K=\hat{D}/(\hat{V}_\infty\cdot\hat{\tau}). \tag{13}$$

Then the nondimensional equations to be used in numerical calculation are

$$\frac{\partial v_0}{\partial r}=-\frac{1}{r}(v_0+u_1)-\frac{v_0}{\rho_0}\frac{\partial \rho_0}{\partial r} \tag{14}$$

$$\frac{\partial p_0}{\partial r}=-\rho_0 v_0\frac{\partial v_0}{\partial r}+\frac{4}{3Re}\frac{\partial^2 v_0}{\partial r^2}$$
$$-\frac{1}{Re}\left\{\frac{2}{r^2}(v_0+u_1)-\frac{1}{r}\frac{\partial v_0}{\partial r}\right\}$$
$$-\frac{1}{3Re}\left\{\frac{1}{r^2}(v_0+u_1)-\frac{1}{r}\left(\frac{\partial v_0}{\partial r}+\frac{\partial u_1}{\partial r}\right)\right\} \tag{15}$$

$$\frac{\partial p_2}{\partial r}=\frac{1}{2}\left\{\rho_0 v_0\frac{\partial v_0}{\partial r}+2\frac{\rho_0 u_1}{r}(u_1+v_0)-\frac{\partial p_0}{\partial r}\right\} \tag{16}$$

$$\frac{\partial u_1}{\partial t}=-v_0\frac{\partial u_1}{\partial r}-\frac{u_1}{r}(u_1+v_0)-\frac{2}{r}\frac{p_2}{\rho_0}$$
$$-\frac{1}{3Re}\frac{1}{r\rho_0}\left(\frac{\partial v_0}{\partial r}+\frac{u_1}{r}+\frac{v_0}{r}\right)$$
$$+\frac{1}{Re}\frac{1}{\rho_0}\left\{\frac{\partial^2 u_1}{\partial r^2}+\frac{1}{r}\frac{\partial u_1}{\partial r}-\frac{2}{r^2}(u_1+v_0)\right\} \tag{17}$$

$$\frac{\partial T_{t,r0}}{\partial t}=-v_0\frac{\partial T_{t,r0}}{\partial r}-KC(T_{t,r0}-T_{v0})$$
$$-(\gamma_f-1)T_{t,r0}\left(\frac{\partial v_0}{\partial r}+\frac{u_1}{r}+\frac{v_0}{r}\right)+\frac{1}{RePr_1}$$
$$\times\frac{1}{\rho_0}\left(\frac{\partial^2 T_{t,r0}}{\partial r^2}+\frac{1}{r}\frac{\partial T_{t,r0}}{\partial r}+\frac{2}{r^2}T_{t,r2}\right)$$
$$+\frac{2\gamma_f}{Re}\left\{\left(\frac{\partial v_0}{\partial r}\right)^2+\left(\frac{u_1+v_0}{r}\right)^2\right\} \tag{18}$$

$$\frac{\partial T_{v0}}{\partial t}=-v_0\frac{\partial T_{v0}}{\partial r}+K(T_{t,r0}-T_{v0})$$
$$+\frac{1}{RePr_{\mathrm{II}}}\frac{1}{\rho_0}\left(\frac{\partial^2 T_{v0}}{\partial r^2}+\frac{1}{r}\frac{\partial T_{v0}}{\partial r}\right) \tag{19}$$

$$p_0=\frac{\gamma_f-1}{\gamma_f}\rho_0 T_{t,r0}. \tag{20}$$

In equations (17)–(19) the unsteady terms are added in the left side, to make convergence of numerical calculations more successful.

3. BOUNDARY CONDITIONS

In the free stream far upstream the stagnation point, all quantities of the flow and temperature fields are assumed to be constant and the nondimensional boundary conditions are given as

$$\rho_0 = 1, \qquad v_0 = -1, \quad u_1 = 1$$
$$T_{t,r0} = T_{t,r\infty}, \quad T_{v0} = T_{v\infty}, \quad p_2 = 0. \tag{21}$$

In connection with equation (21), the relation between the translational–rotational and the vibrational temperatures should be investigated. Two possible cases are taken up in this study: equilibrium and nonequilibrium cases. In the equilibrium case $T_{t,r\infty} = T_{v\infty}$. In the other case, which is a nonequilibrium case, the temperature nonequilibrium is caused by a shock wave and $T_{t,r\infty} > T_{v\infty}$ is assumed as the boundary condition in the free stream.

On the cylinder surface, the velocity slip u_1 and the temperature jumps are considered. The relations to indicate these conditions are:

$$v_0 = 0,$$
$$u_1 = \frac{2-f}{f} Kn \left(\frac{\partial u_1}{\partial r}\right)_{r=0.5} \tag{22}$$

$$T_{t,r0} = T_w + \frac{2-\alpha_{t,r}}{\alpha_{t,r}} \frac{2}{\gamma_f + 1} \frac{Kn}{Pr_{\mathrm{I}}} \left(\frac{\partial T_{t,r0}}{\partial r}\right)_{r=0.5} \tag{23}$$

$$T_{v0} = T_w + \frac{2-\alpha_v}{\alpha_v} \cdot \frac{Kn}{Pr_{\mathrm{II}}} \left(\frac{\partial T_{t,r}}{\partial r}\right)_{r=0.5}. \tag{24}$$

4. NUMERICAL CALCULATION AND RESULTS

The differential equations to be solved are equations (14)–(19) constructing a set of nonlinear first and second order equations. The numerical procedure to solve these equations by use of equation (20) is as follows. First, appropriate initial profiles are assumed for u_1, $T_{t,r0}$ and T_{v0} to solve unsteady differential equations (17)–(19) for them by integrations with time. In the process of these integrations, the values of v_0, p_0 and p_2, which are obtained from integrating equations (14)–(16) and using previous values of u_1 and ρ_0, are used where ρ_0 is calculated from equation (20) in advance. Then the same calculation procedures are continued until the steady solutions are obtained. In the calculations two typical values for Reynolds number are taken up in consideration of the effect of $\hat{T}_{t,r2}$ and future comparisons with experiments by use of fine wire and a small cylinder; a low Re region of 10–100 and the other Re region of 10^4. In numerical calculations for both Re regions, as the physical properties, those of CO_2 are assumed, because not only the properties, but also the vibrational relaxation time is well known. The coefficient for velocity slip f and the accommodation coefficient $\alpha_{t,r}$ for the translational–rotational temperature jump are assumed to be a unity, but the accommodation coefficient for the vibrational temperature α_v is taken as 0.001, taking into account the report by Schubert [5] on N_2. The adaptability of this assumption should be checked in comparison with experiments in future.

Before proceeding to show numerical results, a general discussion of the influence of vibrational relaxation on the heat transferred to the surface is made. The following steady state equation for $T_{t,r0}$ is obtained from equations (14), (18) and (20).

$$v_0 \frac{\mathrm{d}T_{t,r0}}{\mathrm{d}r} = -K \frac{C}{\gamma_f} (T_{t,r0} - T_{v0}) + \frac{v_0}{\rho_0} \frac{\mathrm{d}p_0}{\mathrm{d}r}$$
$$+ \frac{1}{RePr_{\mathrm{I}}\gamma_f} \frac{1}{\rho_0} \left(\frac{\mathrm{d}^2 T_{t,r0}}{\mathrm{d}r^2} + \frac{1}{r} \frac{\mathrm{d}T_{t,r0}}{\mathrm{d}r}\right)$$
$$+ \frac{2}{Re} \left\{ \left(\frac{\mathrm{d}v_0}{\mathrm{d}r}\right)^2 + \left(\frac{u_1 + v_0}{r}\right)^2 \right\}. \tag{25}$$

The steady state equation for T_{v0} is as given by the following equation from equation (19)

$$v_0 \frac{\mathrm{d}T_{v0}}{\mathrm{d}r} = K(T_{t,r0} - T_{v0})$$
$$+ \frac{1}{RePr_{\mathrm{II}}} \frac{1}{\rho_0} \left(\frac{\mathrm{d}^2 T_{v0}}{\mathrm{d}r^2} + \frac{1}{r} \frac{\mathrm{d}T_{v0}}{\mathrm{d}r}\right). \tag{26}$$

A mean temperature T_m is defined by the next equation from $T_{t,r0}$ and T_{v0}

$$T_m = \frac{\hat{C}_{pt,r}}{\hat{C}_p} T_{t,r0} + \frac{\hat{C}_{vv}}{\hat{C}_p} T_{v0}. \tag{27}$$

It is well assumed that $\hat{\lambda}_{t,r}/\hat{\lambda}_v$ is proportional to $\hat{C}_{pt,r}/\hat{C}_{vv}$ and we have

$$\lambda_{t,r} = \hat{\lambda}_{t,r}/(\hat{\lambda}_{t,r} + \hat{\lambda}_v) = \hat{C}_{pt,r}/\hat{C}_p,$$
$$\lambda_v = \hat{\lambda}_v/(\hat{\lambda}_{t,r} + \hat{\lambda}_v) = \hat{C}_{vv}/\hat{C}_p. \tag{28}$$

By use of equation (28) the following approximate relation for T_m is reduced from equations (25) and (26)

$$v_0 \frac{\mathrm{d}T_m}{\mathrm{d}r} = \frac{\gamma_f}{\gamma_f + C} \frac{v_0}{\rho_0} \frac{\mathrm{d}p_0}{\mathrm{d}r}$$
$$+ \frac{1}{RePr} \frac{1}{\rho_0} \left(\frac{\mathrm{d}^2 T_m}{\mathrm{d}r^2} + \frac{1}{r} \frac{\mathrm{d}T_m}{\mathrm{d}r}\right)$$
$$+ \frac{\gamma_f}{\gamma_f + C} \frac{2}{Re} \left\{ \left(\frac{\mathrm{d}v_0}{\mathrm{d}r}\right)^2 + \left(\frac{u_1 + v_0}{r}\right)^2 \right\}. \tag{29}$$

In equation (29), the parameter K is not explicitly included and T_m is obtained from equation (29) to satisfy the boundary conditions. The nondimensional heat transferred to the surface is defined as follows and got from equation (30):

$$Q = \lambda_{t,r} \left(\frac{\mathrm{d}T_{t,r0}}{\mathrm{d}r}\right)_{r=0.5} + \lambda_v \left(\frac{\mathrm{d}T_{v0}}{\mathrm{d}r}\right)_{r=0.5}. \tag{30}$$

Equation (30) is also shown as follows by use of the approximate relation (28):

$$Q = \left(\frac{\mathrm{d}T_m}{\mathrm{d}r}\right)_{r=0.5}. \tag{31}$$

The mean temperature at the surface is given as follows from equations (23) and (24)

Table 1. Wall temperature and free stream translational–rotational and vibrational temperatures and K in low Reynolds number case ($Re = 50$)

	Calculated case No.	$\hat{T}_w$(K)	$\hat{T}_{t,r\infty}$(K)	$\hat{T}_{v\infty}$(K)	K
Equilibrium	1	400	400	400	1.0
	2	400	400	400	10^{-4}
Nonequilibrium	3	400	500	400	1.0
	4	400	500	400	10^{-4}

Table 2. Nondimensional temperature gradient and heat fluxes at the wall in low Reynolds number case ($Re = 50$)

	Calculated case no.	$(\mathrm{d}T_{t,r0}/\mathrm{d}r)_{r=0.5}$	$(\mathrm{d}T_{v0}/\mathrm{d}r)_{r=0.5}$	Nondimensional heat flux, Q	$\Delta Q/Q$
Equilibrium	1	3.436	3.372×10^{-3}	2.529	$(Q_2-Q_1)/Q_1 = 15.1\%$
	2	3.957	9.321×10^{-7}	2.912	
Nonequilibrium	3	5.232	8.035×10^{-3}	3.853	$(Q_4-Q_3)/Q_3 = 81.0\%$
	4	9.476	4.650×10^{-6}	6.973	

$$T_m = T_w + C_1 \frac{2-\alpha_{t,r}}{\alpha_{t,r}} Kn \left(\frac{\mathrm{d}T_m}{\mathrm{d}r}\right)_{r=0.5} + \left(1 - \frac{C_1}{C_2}\,\frac{2-\alpha_{t,r}}{2-\alpha_v}\,\frac{\alpha_v}{\alpha_{t,r}}\right)\frac{C}{\gamma_f + C}(T_{v0} - T_w)_{r=0.5} \tag{32}$$

where C_1 and C_2 are given respectively as follows:

$$C_1 = \frac{2}{\gamma_f + 1}\,\frac{1}{Pr_{\mathrm{I}}}$$
$$C_2 = \frac{1}{Pr_{\mathrm{II}}}. \tag{33}$$

The third term in the right side of equation (32) represents the influence of vibrational relaxation. By investigation equation (32), it is understood and explained in details in the numerical results described later that for small K the vibrational temperature is almost frozen and equal to the main stream temperature and for K being an order of unity the temperature difference between the translational–rotational degree and the vibrational degree gets small and for large K the vibrational temperature becomes equal to the translational and rotational temperature. When α_v is much smaller than unity, $(T_{v0} - T_w)_{r=0.5}$ is not zero but finite, and as seen from equation (32), $(\mathrm{d}T_m/\mathrm{d}r)_{r=0.5}$ is affected remarkably by the magnitude of K and consequently the transferred heat is influenced appreciably by the vibrational relaxation. A detailed quantitative assessment of vibrational relaxation on heat transfer is described in the following numerical results.

4.1 *Low Reynolds number cases*

For the cylinder diameter of 5 μm, the main stream velocity $\hat{V}_\infty = 280$ m/s, the main stream pressure $\hat{P}_\infty = 0.5$ atm, Reynolds number $Re = 50$, and Knudsen number $Kn = 2.64 \times 10^{-2}$, numerical calculations were made for thermal equilibrium and nonequilibrium cases by taking the ratio K of the characteristic time of flow $\hat{D}/\hat{V}_\infty$ to the relaxation time $\hat{\tau}$ as a parameter. The cases calculated are shown in Table 1.

(1) *Equilibrium case.* In Figs. 2 and 3 the variations of pressure, density, translational–rotational temperature and vibrational temperature along the stagnation stream line are shown against the nondimensional distance from the center of the cylinder. The position of $r = 0.5$ represents the surface. The solid lines show the results for $K = 1.0$ and the broken line for $K = 10^{-4}$ when the vibrational degree is frozen. In Table 2, the temperature gradients and the nondimensional heat fluxes calculated by equation (30) are shown. Generally the heat fluxes at the surface depend on K and

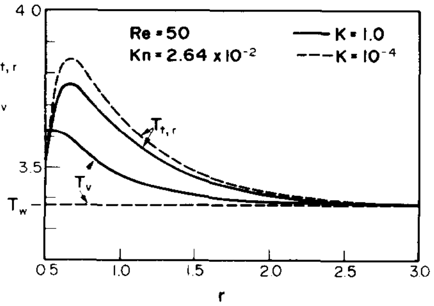

FIG. 2. Temperature profiles along the stagnation stream line for equilibrium case.

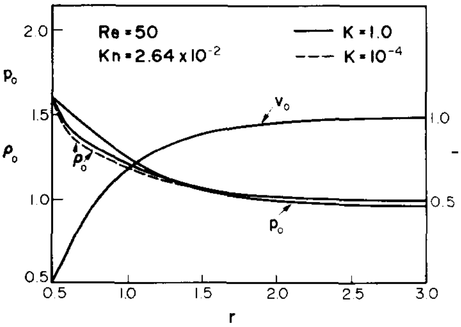

FIG. 3. Profiles of radial velocity, pressure and density along the stagnation stream line for equilibrium case.

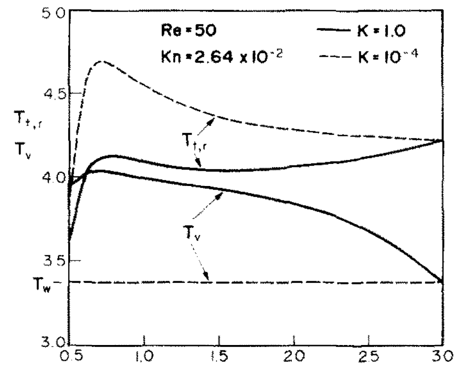

FIG. 4. Temperature profiles along the stagnation stream line for nonequilibrium case.

the heat flux for the vibrationally frozen case is lower than that for the equilibrium case. However in case of $T_\infty = T_w$, for smaller K the heat transferred to the vibrational degree is small, and the heat caused by aerodynamic heating is almost supplied to the translational and rotational degrees. Consequently, $T_{t,r}$ has its high peak value and its gradient of $\hat{T}_{t,r}$ at the surface is higher. On the other hand, for K being equal to unity, the heat transferred to the vibrational degree is large and the peak value of $T_{t,r}$ becomes smaller and the gradient of $\hat{T}_{t,r}$ at the surface decreases. Due to the condition that $\alpha_{t,r} \gg \alpha_v$ at the surface the gradient of $T_{t,r}$ is much larger than that of $\hat{T}_v$, therefore the heat flux at the surface is chiefly controlled by the gradient of $\hat{T}_{t,r}$ and the heat flux for small K is higher than that for large K.

(2) *Nonequilibrium case.* To bring about a nonequilibrium state between the translational–rotational temperature and the vibrational temperature, it is assumed that a shock wave stands at $r = \hat{r}/\hat{D} = 3.0$ upstream of the stagnation point. The conditions taken up in the numerical calculation for the nonequilibrium case are also shown in Table 1 as the examples of Nos. 3 and 4 having different values of K. In Figs. 4 and 5 the results concerning the distributions along the stagnation stream line are shown with K as parameter and in Table 2 the temperature gradient and heat flux at the surface are listed. The heat flux depends on the value of K and is larger for smaller K. The difference between the heat fluxes for these values of K is bigger in the nonequilibrium case than in the equilibrium case. The dependence of the heat flux on K is caused by the large temperature difference between the translational–rotational degree and the vibrational degree. When a nonequilibrium of temperatures exists in the free stream, the temperature difference increases with the decrease of K as compared with the equilibrium case. On the other hand, for large K, the temperature difference decreases when approaching the stagnation point notwithstanding the temperature nonequilibrium in the free stream, and the tendency is rather the same with the equilibrium case. As described above, in the nonequilibrium case of the free stream the temperature difference between the translational–rotational and the vibrational degrees is accelerated with the decrease of the value of K and consequently the heat flux at the surface increases with the decrease of K.

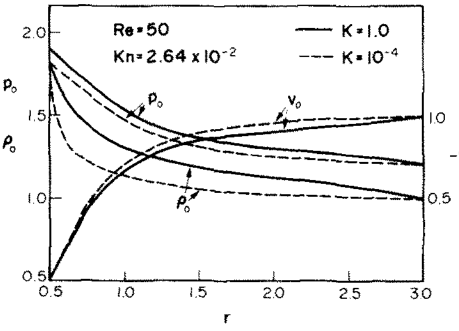

FIG. 5. Profiles of radial velocity, pressure and density along the stagnation stream line for nonequilibrium case.

4.2 *High Reynolds number cases*

Discussing a flow around a cylinder of 1 mm dia having $\hat{V}_\infty = 280$ m/s, $\hat{P}_\infty = 0.5$ atm, Re $\doteqdot 10^4$, and $Kn = 1.32 \times 10^{-4}$, numerical calculations were carried on taking K as a parameter, and following the same procedure as those used in Section 4.1. The conditions used in the calculations are listed in Table 3.

In Figs. 6 and 7, the calculated distributions along the stagnation stream line of pressure, density, velocity, translational–rotational temperature and vibrational temperature are shown. In Fig. 8 the extended distributions of temperatures shown in Fig. 6, and in Table 3 the temperature gradients and the heat fluxes at the wall obtained from equation (30) are also represented. In these cases the gradients of $T_{t,r}$ at the wall are very large compared with T_v and the heat flux depends much on K. This is understood for the following reason. In the vicinity of the surface, when the energy transfer between translational–rotational degrees and vibrational degree is violate but α_v is much smaller than unity, T_v has a slip at the surface notwithstanding Kn is of very small value. This is due to the fact that heat is not transferred to the surface by thermal conduction of vibrational temperature and that some heat is transferred to translational and rotational temperature and then transferred to the surface. This results in an adiabatic condition for the vibrational degree, even though the translational and rotational degrees are in thermal equilibrium with the surface, and an effect of slip condition at the surface is appreciable even though Kn is much smaller than unity. Thus it is found that in case of appreciable vibrational relaxation and α_v being much smaller than unity, the effect of vibrational temperature jumps at the surface is still remarkable and the vibrational relaxation phenomenon gives much influence on the temperature field even in a continuum flow.

Table 3. Wall temperature, free stream temperatures, nondimensional numerical results temperature gradient and heat flux in high Reynolds number case ($Re = 10^4$)

Calculated case no.	$\hat{T}_w$ (K)	$\hat{T}_{t,r\infty}$ (K)	$\hat{T}_{v\infty}$ (K)	K	$(\mathrm{d}T_{t,r0}/\mathrm{d}r)_{r=0.5}$	$(\mathrm{d}T_{v0}/\mathrm{d}r)_{r=0.5}$	Q	$\Delta Q/Q$
5	400	400	400	1.0	43.04	7.646×10	31.87	$(Q_6-Q_5)/Q_5$
6	400	400	400	10^{-4}	54.78	3.114×10^{-4}	40.31	$=26.5\%$

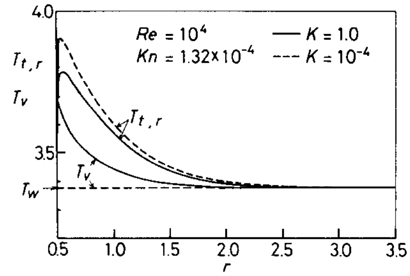

FIG. 6. Temperature profiles along the stagnation stream line for high Reynolds number case.

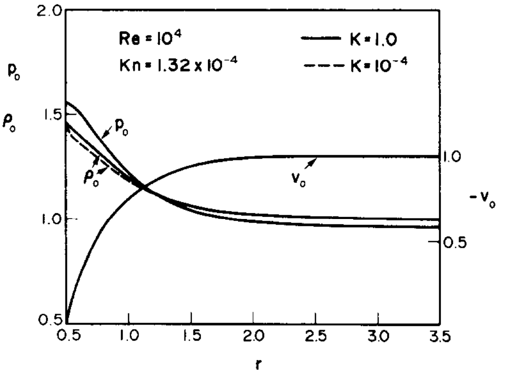

FIG. 7. Profiles of radial velocity, pressure and density along the stagnation stream line for high Reynolds number case.

4.3 *Dependency of transferred heat on vibrational relaxation*

Figure 9 shows dependency of heat transferred to the surface at the stagnation point on K by taking the value for $K = 10^{-4}$ as unity. The solid line shows the result for $Re = 50$ and the broken line for $Re = 10^4$. In the both cases, the transferred heat is the minimum when K is of about unity. In the analysis for these results T_∞ is taken equal to T_w, and the heat transferred to the surface is originally caused by aerodynamic heating, and subsequently by the energy transfer between the internal degrees of molecules. The thermal energy brought about by aerodynamic heating is supplied to the translational–rotational degrees and then partly transferred to the vibrational degree from the former degrees. The energy transferred to the translational–rotational degrees is finally transferred to the surface, while most of the energy introduced to the vibrational degree is transferred to the surface through the translational–rotational degrees as is described in the previous section. Consequently for small K, that is the almost frozen case, the heat caused by aerodynamic heating is transferred effectively to the surface, because most of it is directly transferred to the translational–rotational degrees. On the other hand, for large K, that is in the nearly equilibrium case, transfer of heat caused by aerodynamic heating is also remarkable, because the heat transferred to the vibrational degree is mostly transferred to the translational–rotational degrees before gas particles reach the surface and most of the energy is transferred

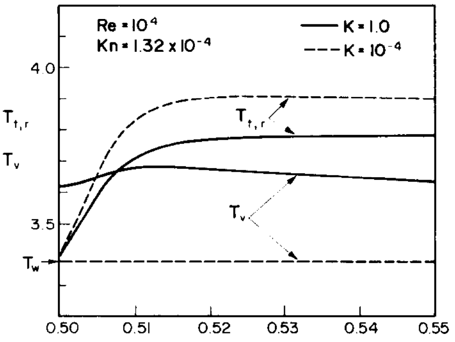

FIG. 8. Coordinate extended profile of temperatures shown in Fig. 6.

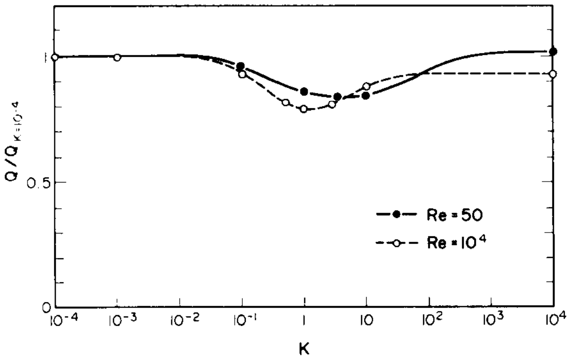

FIG. 9. Dependency of transferred heat at the stagnation point on K, when taking the value for $K = 10^{-4}$ as unity.

to the surface through the latter degrees.

When K is about a unity, only a portion of energy transferred to the vibrational degree is transferred to the translational–rotational degrees and a finite energy is contained in the vibrational degree of molecule when the gas particles collide with the surface. This is seen as $(T_{v0} - T_w)_{r=0.5}$ is large and heat transferred to the surface is small for K being of about unity.

In this paper most of numerical calculations were done for $T_w = T_\infty$ to make clear the mechanism of energy transfer between internal degrees of molecule. In case of $T_w < T_\infty$, where heat transferred to the surface is much influenced by the temperature difference between the wall and the gas, dependency of the transferred heat on K is expected to be different from the case of $T_w = T_\infty$. For small K, heat transferred to the surface is small, because the energy in the vibrational degree is hardly transferred to the translational–rotational degrees, because gas particles of multi-atomic molecule having much energy in the vibrational degree collide with the surface without giving a remarkable contribution of vibrational energy to heat flux at the surface due to small α_v. On the other hand, for large K, heat transferred to the surface is large, because the energy contained in the vibrational degree is mostly transferred to the translational–rotational degrees before gas particles collide with the surface, and is transferred to the surface through the translational–rotational degrees. Therefore, heat transferred to the surface is expected to increase with the value of K. This dependency of transferred heat on K is different from that for $T_w = T_\infty$ which is shown in Fig. 9. This discussion should be investigated by experiments in future.

5. CONCLUSIONS

The effects of vibrational relaxation of multi-atomic gases of subsonic flow on heat transfer in the vicinity of the stagnation of a cylinder have been studied and the following conclusions have been obtained.

(1) The total heat transferred to the surface is the sum of heats due to the translational–rotational temperature and the vibrational temperature. In case of α_v being much smaller than unity, the transferred heat from the vibrational degree is mostly through the translational–rotational degrees. Consequently the ratio of the heats depends much on the ratio of the flow characteristic time to the relaxation time of molecule, and the total heat is much influenced by this ratio of the characteristic times.

(2) In the case of the temperature nonequilibrium in the free stream, the dependence of heat flux on the ratio of the flow to the vibrational relaxation time is more remarkable than in the equilibrium case.

(3) Even in a flow region of Kn being about 10^{-4} which is considered as continuum, the effect of vibrational relaxation on transferred heat is appreciable under condition of α_v being much smaller than unity.

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LES EFFETS DE LA RELAXATION VIBRATIONNELLE DES MOLECULES MULTIATOMIQUES SUR LE TRANSFERT THERMIQUE AU POINT D'ARRET

Résumé—On étudie théoriquement l'écoulement et le champ de température d'un gaz multiatomique près du point d'arrêt d'un cylindre, en tenant compte de la relaxation de vibration des molécules et en mettant l'accent sur le transfert thermique. A partir des équations du milieu continu, les sauts de vitesse et de température sont considérés avec les conditions aux limites et les effets de la relaxation vibrationnelle sur l'écoulement subsonique sont discutés. Le flux de chaleur total transféré à la surface est la somme des chaleurs dues à la température de translation–rotation et à la température de vibration. Le rapport de ces chaleurs dépend beaucoup du rapport du temps caractéristique de l'écoulement au temps de relaxation moléculaire et il en est de même pour le flux total. L'effet du temps de relaxation sur le transfert thermique est plus remarquable dans le cas de l'équilibre que dans celui du déséquilibre vibrationnel. Même pour un écoulement continu à nombre de Knudsen d'environ 10^{-4}, l'effet de la relaxation vibrationnelle sur le transfert thermique est encore appréciable.

EINFLÜSSE DER VIBRATIONSRELAXATION VON VIEL-ATOMIGEN MOLEKÜLEN AUF DEN WÄRMEÜBERGANG AM STAUPUNKT

Zusammenfassung—Es wird das Geschwindigkeits- und Temperaturfeld eines viel-atomigen Gases in der Nähe des Stagnationsbereichs eines Zylinders theoretisch untersucht unter Berücksichtigung molekularer Vibrationsrelaxation wobei besonderer Wert auf Wärmeübergangsvorgänge gelegt wird. Aufgrund der Erhaltungssätze werden die Geschwindigkeits- und Temperatursprünge der Translations- Rotations- und Vibrations- freiheitsgrade betrachtet, während die Randbedingungen und Einflüsse der Vibrationsrelaxation für Unterschallströmung diskutiert werden. Die gesamte an die Oberfläche übertragene Wärme ergibt sich als die Summe der Wärmen aus Translations- Rotations-Temperatur und Vibrations-temperatur. Das Verhältnis der Wärmen hängt stark ab vom Verhältnis der Zeit der Strömungscharakteristik zur Relaxationszeit der Moleküle und damit ist die Gesamtwärme stark beeinflußt durch dieses Verhältnis von charakteristischen Zeiten. Der Einfluß der Strömungsrelaxations- zeit auf den Wärmeübergang ist im thermischen Gleichgewicht ausgeprägter als im Nichtgleichgewicht des Vibrationshauptstromes. Selbst im Continuumstrom bei Knudsen-Zahlen von ungefähr 10^{-4} ist der Effekt der Vibrations-Relaxation auf den Wärmeübergang im Stagnationsbereich noch merklich.

ВЛИЯНИЕ КОЛЕБАТЕЛЬНОЙ РЕЛАКСАЦИИ МНОГОАТОМНЫХ МОЛЕКУЛ НА ТЕПЛОПЕРЕНОС В КРИТИЧЕСКОЙ ТОЧКЕ

Аннотация — Проведено теоретическое исследование гидродинамического и температурного полей многоатомного газа в области критической точки цилиндра с учетом колебательной релаксации молекул. Особое внимание обращено на характеристики теплообмена. С помощью уравнений неразрывности определялись мгновенные изменения скорости и температуры поступательных, вращательных и колебательных степеней свободы с учетом граничных условий и эффектов колебательной релаксации при дозвуковом течении. Общее количество тепла, поглощенного поверхностью, состоит из суммы тепловых потоков, вызванных поступательно-вращательной и колебательной температурами. Их соотношение сильно зависит от отношения характеристического времени потока к времени релаксации молекул, а, следовательно, суммарный тепловой поток к поверхности сильно зависит от отношения характеристических времен. Влияние времени релаксации потока на теплоперенос больше проявляется при наличии теплового равновесия основного потока, чем при колебательно неравновесном течении. Влияние колебательной релаксации на теплообмен в критической точке довольно заметно даже в потоке с числом Кнудсена, равном примерно 10^{-4}.